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Modelling and forecasting the volatility dynamics of payments transactions in Canadian payments systems

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Modelling and Forecasting the Volatility Dynamics of Payments Transactions in Canadian Payments Systems *

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Abstract

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In this paper, the GARCH-type models are applied to model and forecast the volatility dynamics of the return series of both the total payments volume and total payments value delivered and received at Canada's major payment systems. Based on three typical GARCH-type volatility models, the GARCH model, the EGARCH model, and the GJRGARCH model, we find strong empirical evidence of the volatility clustering and leverage effects in the returns series of total payments volume or total payments value, i.e., large changes in the total payments volume or the total payments value tend to cluster together, and negative shocks could cause higher volatility than positive shocks. Furthermore, we find that all the three GARCH-type volatility models have better forecasting performance than the homoscedastic volatility model. In particular, among the three GARCH-type volatility models, the EGARCH model performs best to forecast the volatility of the returns of the total payments volume or total payments value.

In view of the fact that both the total payments volume and total payments value in payment transactions are important proxies for the potential to pose a range of risks, such as credit risk, liquidity risk, and operational risk, for the transaction counterparties, our work provides empirical evidence that the GARCH-type models can be further used to perform a wide range of downside risk analysis of the total payments volume or the total payments value at Canada's main payments system.

Keywords: Total payments volume, total payments value, conditional mean, volatility clustering, leverage effects, the impact of news on volatility, downside risk.

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1 Introduction

Payment transactions could generate a range of risks for counterparties that undertake the transactions, the intermediaries that process the transactions, and the central banks through which final interbank settlement occurs. Both the payments volume and payments value in payment transactions are important proxies for the possibility to pose credit risk, liquidity risk and operational risk, and they reflect the magnitude and potential impact on the economy as a whole and financial system in the event of disruption. Therefore, both the payments volume and payments value provide useful information for monitoring the economic development and stability of the financial system, promoting the efficient use of financial resources, and facilitating the conduct of monetary policy.

At the Payments Canada, the forecasting model used for the total payments volume and total payments value is an autoregressive integrated moving average (ARIMA) model. The error terms in the ARIMA model are generally assumed to be independently, identically distributed variables sampled from a given distribution, a normal distribution or a t-distribution. The basic assumption of this ARIMA model is that the expected value of all error terms when squared is the same at any given time, which indicates that the error terms in this model exhibit constant volatility. However, the volatility of the error terms may not be a constant and the error terms may reasonably be expected to be larger for some points or ranges of the data than for other, that is the error terms could follow from heteroscedasticity. Volatility plays an important role in characterizing the dynamics of the total payments volume or total payments value, and thereby the volatility predictability is a vital aspect to obtain accurate forecasting for the total payments volume or the total payments value.

In the field of finance, it is a well-known fact that volatility of asset returns is time-varying and predictable (Andersen and Bollerslev, 1998). Volatility of financial asset returns has some common characteristics. In particular, three of the most prominent stylized facts about volatility are that volatility exhibits persistence; volatility is mean reverting; and the shocks to asset returns have an asymmetric impact on volatility (Engle and Patton, 2001). The volatility persistence, also referred to as volatility clustering, suggests that volatility exhibits periods of relative calm followed by more turbulent periods. That volatility is mean reverting to some extent suggests that volatility is not diverging to infinity but is moving within some range. The volatility's asymmetric dependency of positive and negative shocks is also referred to as the leverage effect, which suggests that negative shocks have a larger impact on the volatility than an equal size positive shock. Many models have been proposed in the literature to model and forecast volatility, and a key aspect in these models is to try to incorporate as many of the three most prominent stylized facts as possible to accurately describe the volatility. In practice, commonly used volatility models include autoregressive conditional heteroscedasticity (ARCH) model developed by Engle (1982) and generalized autoregressive conditional heteroscedasticity (GARCH) model given by Bollerslev (1986). Both the ARCH model and GARCH model are able to model the volatility clustering, but they cannot capture the characteristic that shocks of asst returns have an asymmetric impact on the volatility of the asset returns. To be able to model the asymmetric characteristic and overcome the weaknesses of GARCH model, Nelson (1991) extends the GARCH model to the exponential GARCH (EGARCH) model which is able to allow for asymmetric effects of positive and negative return shocks of asset returns. Glosten, Jagannathan and Runkle (1993) propose the GJRGARCH model which captures the asymmetrical nature of shocks of the asset returns by including a function of the size of shocks of the asset returns.

The above-mentioned GARCH-type volatility models, ARCH model, GARCH model, EGARCH model, and GJRGARCH model, have been applied to model and forecast volatility of a wide range of financial asset returns to conduct financial risk management, asset allocation, and asset pricing (Abdalla, 2012). In this paper, we attempt to use the GARCH-type models to model and forecast the volatility dynamics of the return series of both the payments volume and payments value at Canada's major payment systems, the LVTS and ACSS.

Using the returns series of both the total payments volume and total payments value delivered and received in the LVTS and ACSS, under the assumption that the conditional mean follows an ARMA(1, 1) process, we estimate the homoscedastic volatility model, as well as three other volatility models, which are GARCH(1, 1) model, EGARCH(1, 1) model, and GJRGARCH (1, 1) model. The data are collected in three frequencies, i.e., weekly, monthly, and quarterly frequencies. We find that all the three GARCH-type volatility models have better performance than the homoscedastic volatility model, suggesting that the GARCH-type volatility models significantly improve the estimation and forecast of the volatility of the returns of both the total payments volume and total payment value. In particular, among the three GARCH-type volatility models, the EGARCH model performs best, indicating that not only does volatility clustering plays an important role in forecasting the volatility of the returns of the total payments volume or the total payments value, but also asymmetrical effects of return shocks is a critical factor in capturing the volatility dynamics of the total payments volume or total payments value.

The paper is organized as follows. Section 2 describes the historical payments data used in this paper. Section 3 introduces the volatility models used to estimate and predict the volatility of both total payments volume and total payments value. In Section 4, we describe the estimation method and the in-sample performance of each volatility models. In Section 5, we conduct out-of-sample forecast evaluation of the best in-sample performance relative the basic benchmark model. In Section 6, we provide concluding remarks.

2 Data description

The data consists of both the total payments volume and total payments value delivered and received through the Large Value Transfer System (LVTS) and Automated

Clearing Settlement Systems (ACSS), respectively. The data is collected in three frequencies, i.e., weekly, monthly, and quarterly data. For the time spans, the weekly data covers the period from the first week in January of 2001 to the fifth week in December of 2021; the monthly data covers the period from January 2000 to April 2021; the quarterly data covers the period from the first quarter in 2000 to the forth quarter in 2021.

Figure 1 and Figure 2 plot both the weekly total payments volume and weekly total payments value delivered and received through the ACSS and LVTS, respectively, as well as the weekly returns of total payments volume and total payments value over the period from the first week in January of 2002 to the fifth week in December of 2021, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value. A visual observation of the time series of both total payments volume and total payments value shows that their means and variances are not constant, implying non-stationarity of the time series of both total payments volume and total payments value. Their relative weekly return series seem to be stationary processes with means close to zero but with volatility exhibiting the key characteristic mentioned in the introduction of asset return volatility, volatility clustering, that is large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes of either sign. This suggests that the return series of total payments volume or total payments value exhibit the autoregressive conditional heteroscedasticity, which is prevalent in many financial time series data (Engle, 2001).

Figure 3 and Figure 4 plot the monthly total payments volume and monthly total payments value delivered and received through the ACSS and LVTS over the period from 2000 January to 2021 April, respectively, as well as their monthly returns. A visual observation of both of the Figures shows that the time series of both total payments volume and total payments value are not stationary, but their return series exhibit clearly stationary. The return series in the LVTS given in Figure 4 show periods of high volatility, volatility clustering, as upward movements tend to be followed by other upward movements and downward movements also followed by other downward movements.

Figure 5 and Figure 6 plot the time series of quarterly total payments volume and total payments value in the ACSS and LVTS, respectively, as well as their quarterly returns. As noted that in Figure 5 the volatility clustering feature of the return series of both the total payments volume and total payments value cannot be seen graphically because there is not significantly sustained periods of high or low volatility. However, there is obvious volatility clustering for the return series of both total payments volume and total payments value in the LVTS.

As a next step, we focus on the return series of total payments transactions rather than the levels of the total payments transactions. This is because the return of either total payments volume or total payments value is a complete, scale free summary of total payments transactions, and particularly the return series have more attractive statistical properties that make it be much easier to handle than the total payments transactions. We will take a preliminary look at the summary statistics of the return series of the total payments volume and total payments value in both the ACSS and LVTS.

Table 1 reports the sample size, unconditional mean, unconditional variance, skewness, kurtosis, results of Jargue-Bera test (J.B test) and Augmented Dickey-Fuller test (A.D.F test) of the return series. The mean returns of the three different frequency data are positive and close to zero, which is a common characteristic in financial return series. The kurtosis of the weekly return series in both the ACSS and LVTS is greater than 3, indicating that the weekly return series have very heavy tails showing a strong departure from the Gaussian assumption. [1] The A.D.F test-statistics of all the return series across different data frequencies in Table 1 are less than the critical

[1]Kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution. That is, data sets with high kurtosis tend to have heavy tails, or outliers. Data sets with low kurtosis tend to have light tails, or lack of outlier. The standard normal distribution has a kurtosis of 3.

value (-3.4460) at 5% significant level. Therefore, we reject the null hypothesis that there is a unit root and that these returns series are stationary. For the return series of the total payments transactions in the LVTS for three different frequency data, J.B test rejects the null hypothesis that sample distribution comes from a normal distribution at the 5% significant level (the critical vale = 5.8461). This result is in line with expectations from Figures 2, 4, and 6, which imply that the empirical distributions of the return series of the total payments transactions in the LVTS exhibit significantly heavier tails than the normal distribution. For the weekly return series of both the total payments volume and total payments value in the ACSS, the null hypothesis that sample distribution from a normal distribution is also rejected at 5% significant level by the J.B test.[2]

Figures 7, 8, and 9 show the sample autocorrelation functions for the return series with three different frequencies, suggesting that autocorrelations exist generally. This indicates that a conditional mean model is required for the return series. Although the focus of this paper is the volatility model of the return series (the model for the square root of the conditional variance) rather than the conditional mean model, but for the volatility model to work property, the conditional mean needs to be modeled as well. In this paper, each of the specified volatility models will be used together with the conditional mean model.

[2]The Jarque-Bera test is a two-sided goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test statistic is always nonnegative. If it is far from zero, it signals the data do not have a normal distribution.

Figure 1: Weekly Payments Transactions and Their Weekly Returns in the ACSS

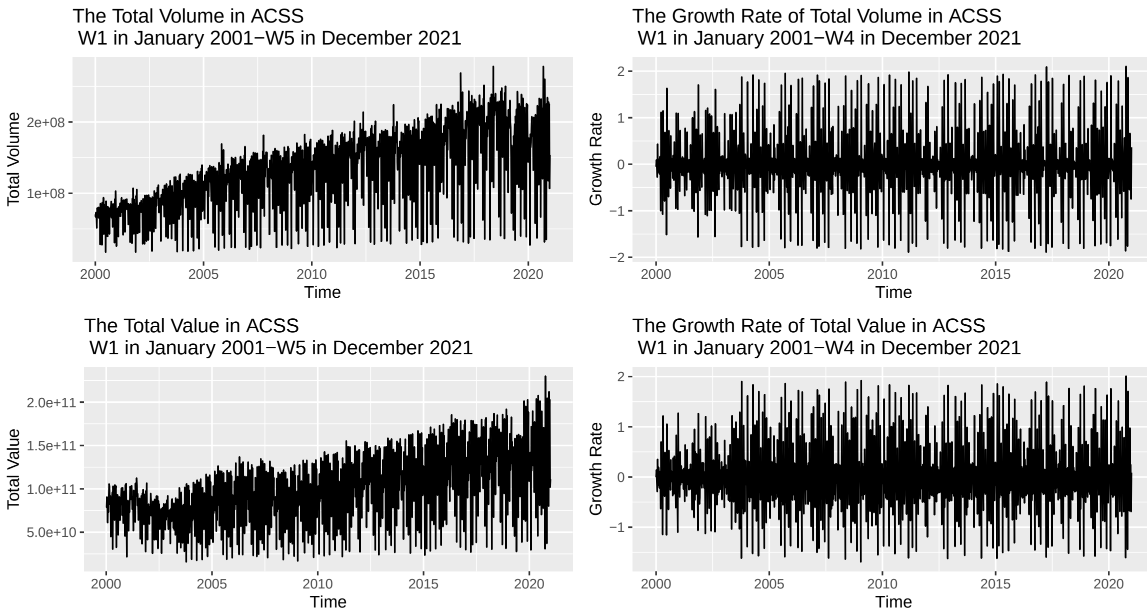

Figure 1 plots the weekly total payments volume and weekly total payments value delivered and received through the ACSS, as well as their weekly returns over the period from W1 in January of 2001 to W5 in December of 2021, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value, and W1, W4, and W5 represent the first week, the fourth week, and the fifth week, respectively.

Figure 2: Weekly Payments Transactions and Their Weekly Returns in the LVTS

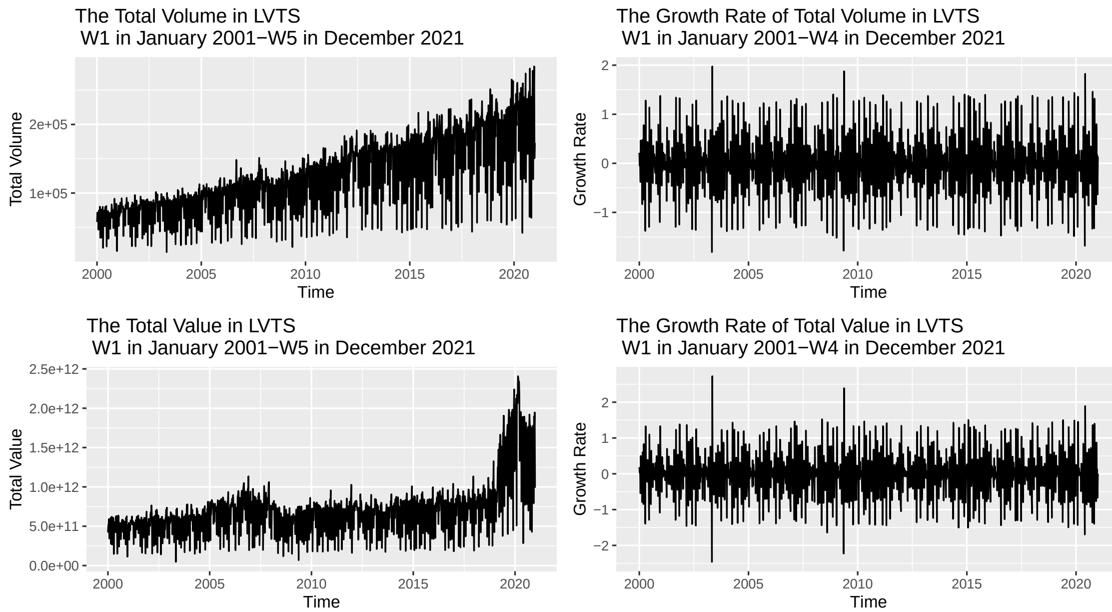

Figure 2 plots the weekly total payments volume and weekly total payments value delivered and received through the LVTS, as well as their weekly returns over the period from W1 in January of 2002 to W5 in December of 2021, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value, and W1, W4, and W5 represent the first week, the fourth week, and the fifth week, respectively.

Figure 3: Monthly Payments Transactions and Their Monthly Returns in the ACSS

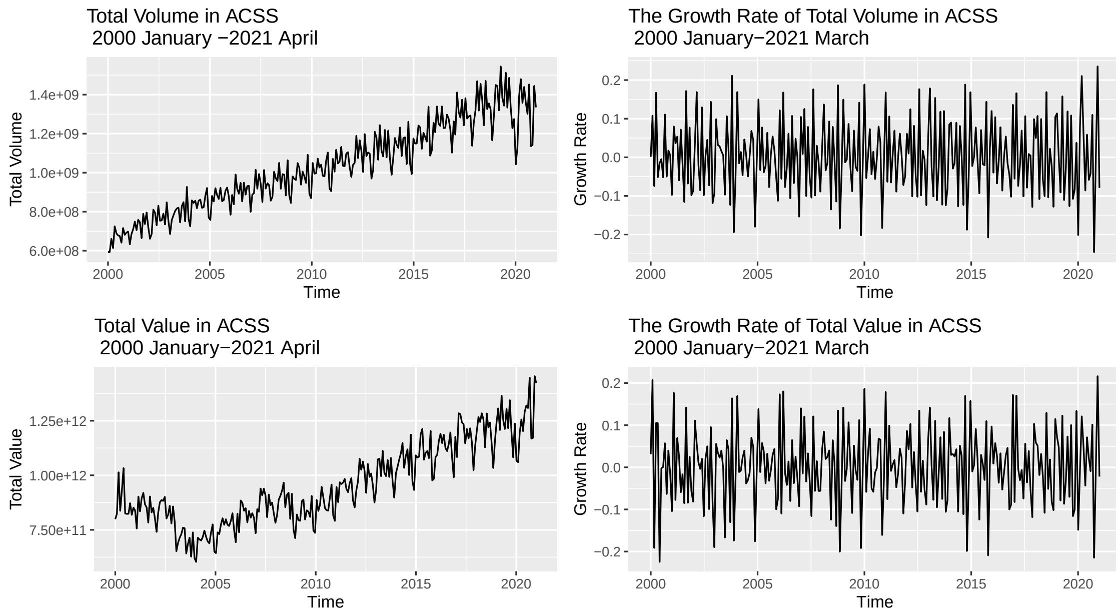

Figure 3 plots the monthly total payments volume and monthly total payments value delivered and received through the ACSS, as well as their monthly returns from January of 2000 April of 2021, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value.

Figure 4: Monthly Payments Transactions and Their Monthly Returns in the LVTS

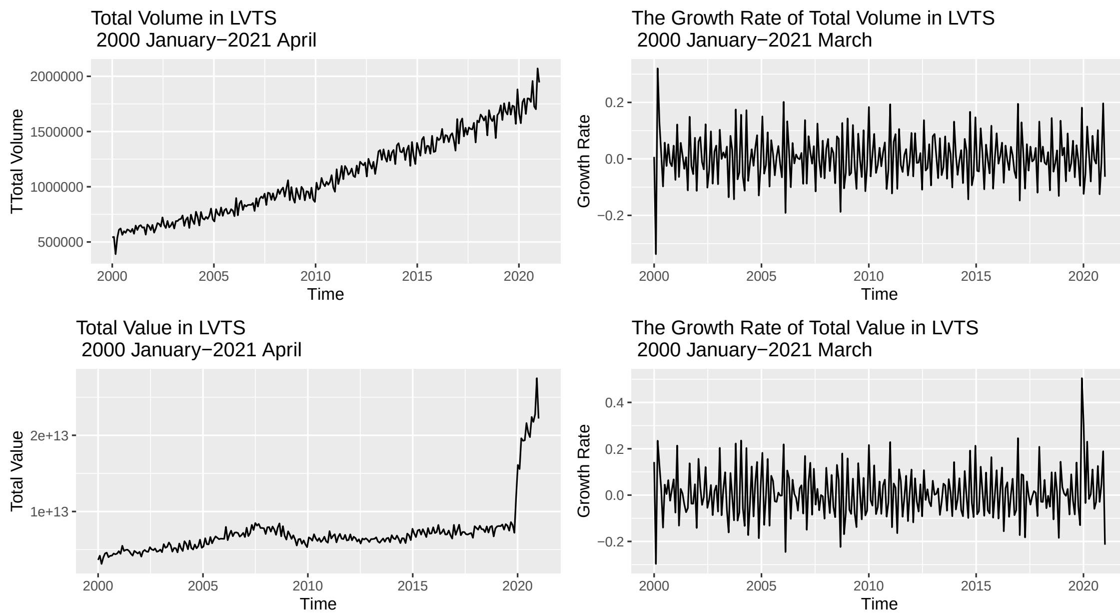

Figure 4 plots the monthly total payments volume and monthly total payments value delivered and received through the LVTS, as well as their monthly returns over the period from January of 2000 to April of 2021, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value.

Figure 5: Quarterly Payments Transactions and Their Quarterly Returns in the ACSS

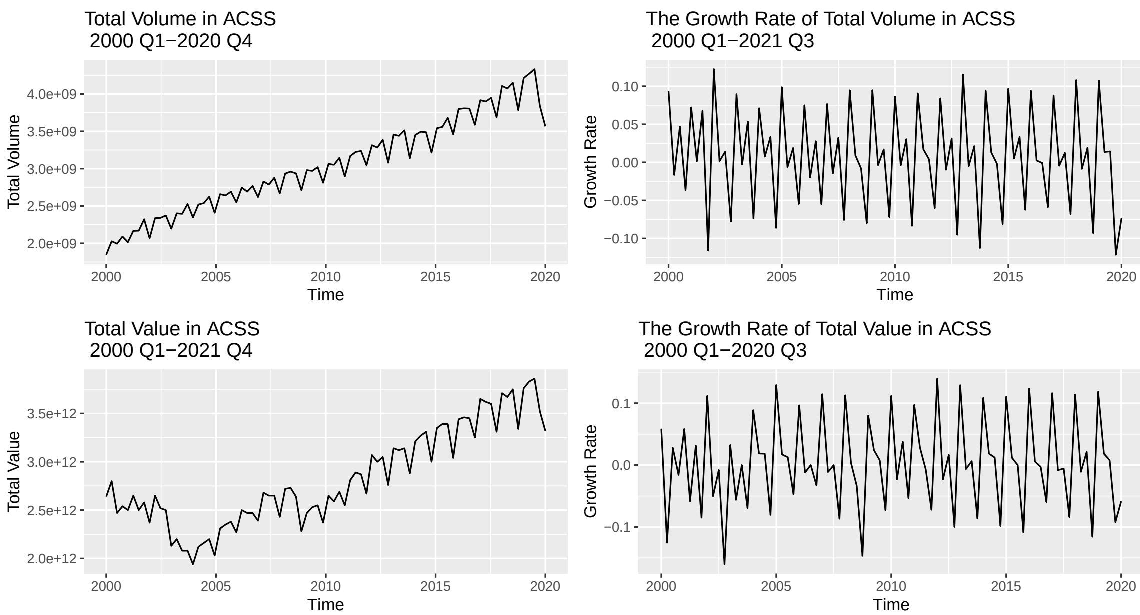

Figure 5 plots the quarterly total payments volume and quarterly total payments value delivered and received through the ACSS, as well as their quarterly returns over the period from 2000Q1 to 2021Q4, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value.

Figure 6: Quarterly Payments Transactions and Their Quarterly Returns in the LVTS

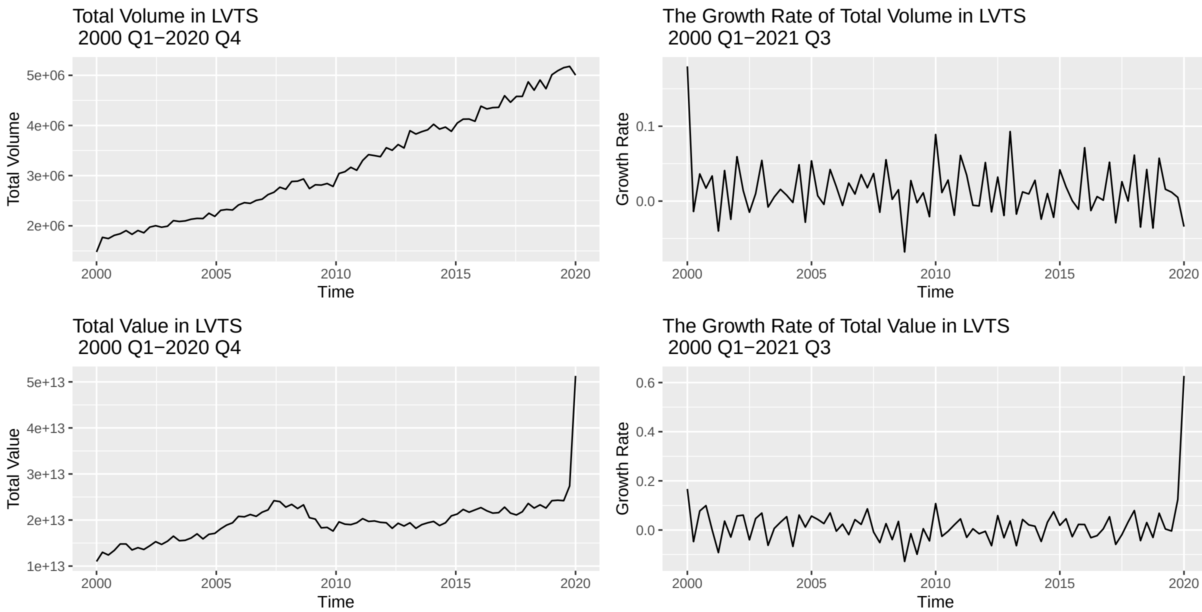

Figure 6 plots the quarterly total payments volume and quarterly total payments value delivered and received through the LVTS, as well as their quarterly returns over the period from 2000Q1 to 2021Q4, where the return series are transformed by taking the first difference of the logarithm of the total payments volume and the total payments value.

Figure 7: Autocorrelation Functions for the Returns of Weekly Payments Transactions

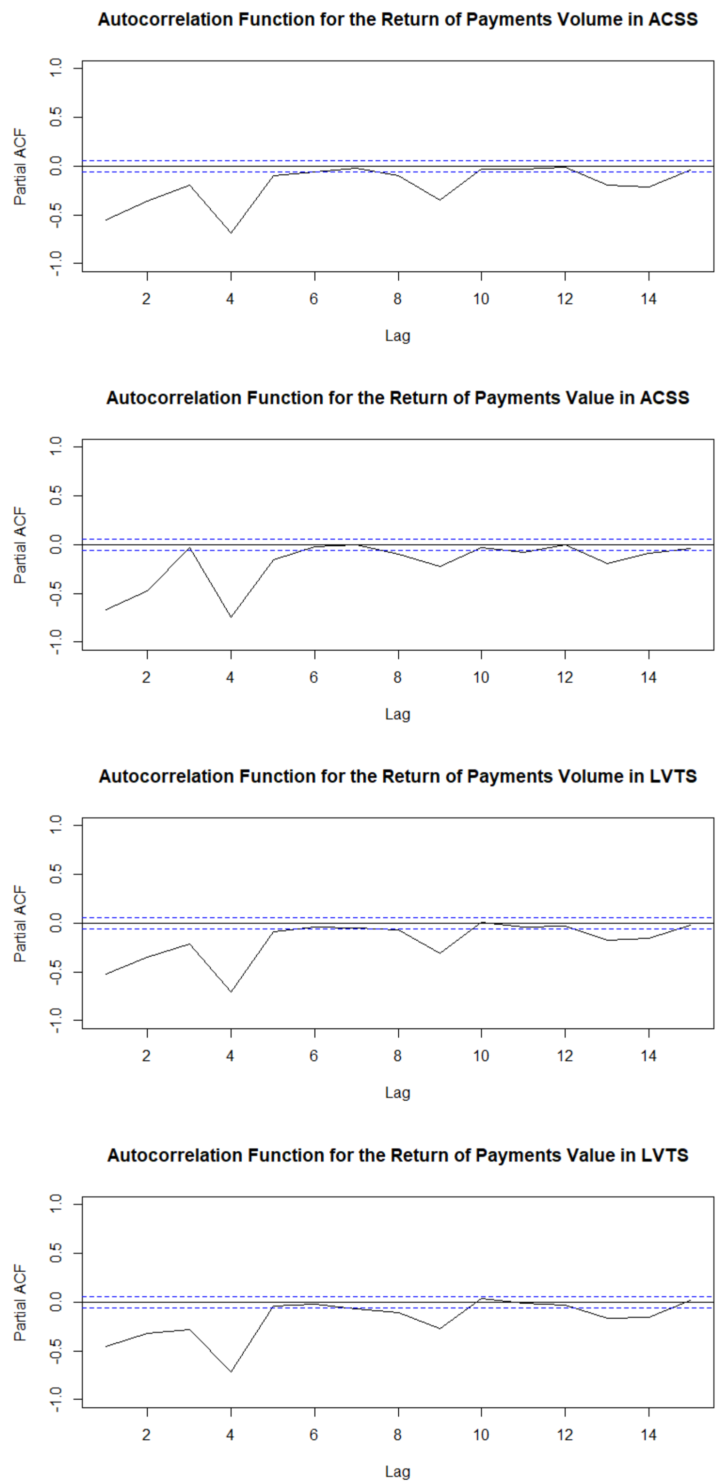

Figure 8: Autocorrelation Functions for the Returns of Monthly Payments Transactions

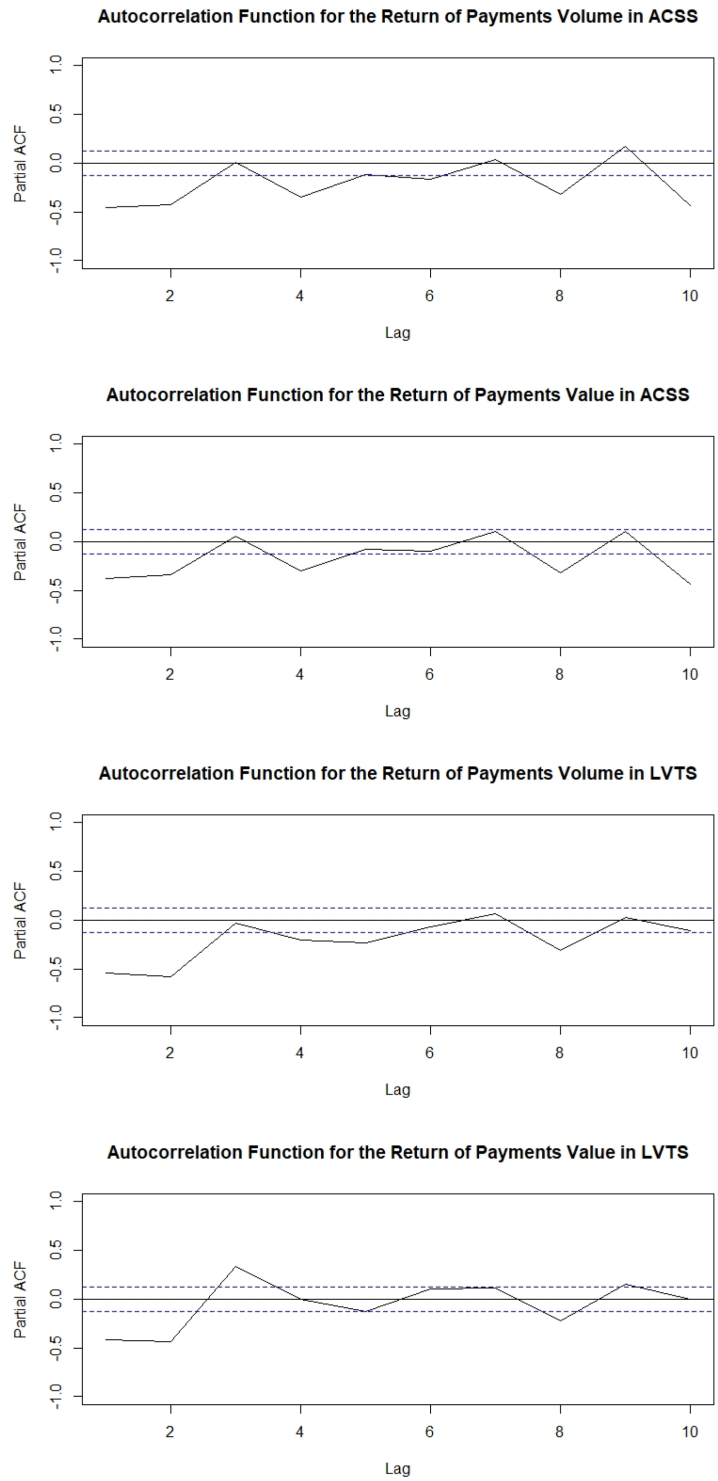

Figure 9: Autocorrelation Functions for the Returns of Quarterly Payments Transactions

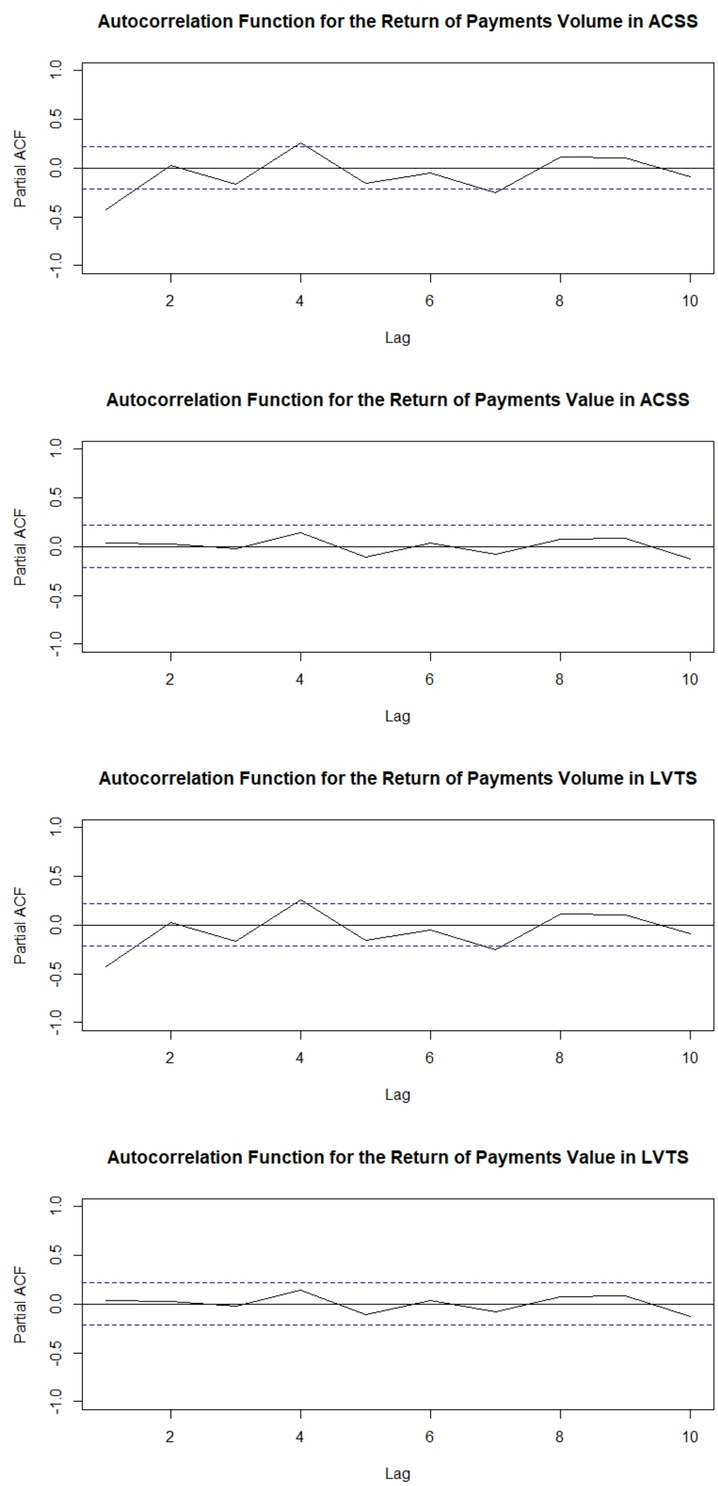

Table 1: **Summary Statistics for the Returns of Total Payments Volume and Total Payments Value**

	Mean	S.D.	Skewness	Kurtosis	J.B Test	A.D.F Test
	ACSS: Weekly return					
Total Volume	0.0006	0.70231	0.1796	4.8034	174.33*	-44.7791*
Total Value	0.0003	0.6643	0.4169	3.9530	82.6950*	-48.696*
	ACSS: Monthly return					
Total Volume	0.0032	0.0958	0.1093	-0.5910	3.9902	-21.292*
Total Value	0.0023	0.0881	-0.0778	2.8094	0.5616	-7.2834*
	ACSS: Quarterly return					
Total Volume	0.0081	0.06389	-0.1156	2.0728	2.7715	-8.2887*
Total Value	0.0028	0.0728	0.0575	0.3175	1.3613	-8.0046*
	LVTS: Weekly return					
Total Volume	0.0009	0.5401	-0.0183	4.6399	138.71*	-43.845*
Total Value	0.0007	0.5706	-0.0387	4.9255	191.28*	-41.34*
	LVTS: Monthly return					
Total Volume	0.0050	0.0868	0.1279	0.5921	5.793*	-28.72*
Total Value	0.0071	0.1088	0.5273	4.0708	24.882*	-21.501*
	LVTS: Quarterly return					
Total Volume	0.0081	0.0639	-0.1156	2.0728	77.37*	-8.2887*
Total Value	0.0028	0.0728	0.0575	2.3175	3084.6*	-8.0046*

Table 1 reports the results of the basic statistical properties of the return of total payments volume and total payments value for ACSS and LVTS in three frequencies, i.e., weekly, monthly, and quarterly frequencies. The time span: the weekly data covers the period from the first week in January of 2001 to the fifth week in December of 2021; the monthly data covers the period from January 2000 to April 2021; the quarterly data covers the period from the first quarter in 2000 to the forth quarter in 2021. J.B. Test refers to the test for normality of the unconditional distribution of returns.

3 Models for forecasting volatility

Volatility is a statistical measurement of the dispersion of returns for a given security or market index. In most cases, the higher the volatility, the riskier the security. Volatility is often measured as either the standard deviation or variance between returns from the same security or market index, and thus it refers to the amount of uncertainty or risk related to the size of changes in a security's value. A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the value of either the security or market index can change dramatically over a short time period in either direction. A lower volatility means that the value of a security or market index does not fluctuate dramatically, and tends to be more steady.

It is a well-known fact that volatility of the returns of a financial asset is time-varying and predictable (Anderson and Bollerslev, 1998). Volatility has some commonly characteristics. In particular, three of the most prominent stylized facts about volatility are that volatility exhibits persistence; volatility is mean reverting; and innovations have an asymmetric impact on volatility (Engle and Patton, 2001). The volatility persistence suggests that volatility exhibits periods of relative calm followed by more turbulent periods and is also referred to as volatility clustering. That volatility is mean reverting to some extent suggests that volatility is not diverging to infinity but is moving within some range. The volatility's asymmetric dependency of positive and negative shocks is also referred to as the leverage effect, which suggests that negative shocks have a larger impact on the volatility than an equal size positive shock. Any model that tries to forecast volatility should be able to incorporate as many of these characteristics as possible to accurately describe the volatility.

The analyses of the data of the total payment transactions in Section 2 suggest some common characteristics. First, the returns of both the total payments volume and total payments value do exhibit some serial correlation of lower order, which suggests that a model is needed for the conditional mean. Second, the return series of both total payments volume and total payments value show the effects of autoregressive conditional heteroscedasticity, which motivates the use of GARCH-type models to capture up the volatility of the return series of the total payments volume or the total payments value. Based on these characteristics, the basic structure for forecasting volatility will be presented in this section, in which we specify the conditional mean model, volatility models, and the error distributions that will be used in the forecasting models.

3.1 Conditional mean

Let r_t stand for the log return series of the total payments volume or total payments value,

$$r_t = log(P_t) - log(P_{t-1}) = log(\frac{P_t}{P_{t-1}}), \tag{1}$$

where P_t is denoted for the total payments volume or the total payment value. We consider first two conditional moments, the conditional mean μ_t and the conditional variance h_t^2, which are defined as,

$$\begin{aligned} \mu_t &= E[r_t|F_{t-1}], \quad (2) \\ h_t^2 &= var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}], \quad (3) \end{aligned}$$

where $E[\cdot|\cdot]$ denotes the conditional expectation operator, and F_{t-1} the information set available at time $t-1$. Let $\varepsilon_t = r_t - \mu_t$, where ε_t is the innovation or residual of r_t, and describes uncorrelated disturbances with zeros mean and plays the role of unpredictable part of the time series r_t. Then we have,

$$r_t = \mu_t + \varepsilon_t. \tag{4}$$

As is evident in the descriptive statistics of the data, the serial dependence of the return series is quite weak and therefore suggests that the conditional mean should be able to be modeled by a relatively simple model (Tsay, 2005). In this paper, the conditional mean is assumed to follow a stationary autoregressive moving average mode, ARMA(1, 1), which is specified as,

$$r_t = c + \theta_1 r_{t-1} + \theta_2 \varepsilon_{t-1} + \varepsilon_t \tag{5}$$

The main object of this paper is the conditional variance of ε_t which is defined as,

$$\varepsilon_t = h_t e_t, \tag{6}$$

where ε_t is the error term; e_t is independently and identically distributed random variable with zero mean and unit variance, but what distribution it should follow needs to be specified; and h_t is the time-varying volatility. We consider two different types of error distributions, the standard normal distribution $e_t \sim N(0,1)$ and the heavier tailed student t-distribution $\sqrt{v/(v-2)}e_t \sim t_v$, where v denotes the number of degrees of freedom. When the error distribution is assumed to be a student t-distribution, the scale factor of e_t is introduced to make the variance of e_t equal to 1. As was evident in the descriptive statistics in Table 1, the return series for weekly, monthly, and quarterly data of the payments transactions in the LVTS, and for weekly data in the ACSS, are typically not normally distributed but display significantly heavier tails than the normal distribution which suggests that the student t-distribution is chosen for the distribution of the error term. However, as showed in Table 1, the normality distribution cannot be rejected by the returns of both the payments volume and payments value with monthly frequency and quarterly frequency in ACSS, the standard normal distribution is specified for e_t.

3.2 Conditional variance

In this paper, the error term, ε_t, is treated as a collective measure of news at time t. A positive ε_t, an unexpected increase in the total payment volume or the total payment value, suggests the arrival of good news, while a negative ε_t, an unexpected decreases in the total payment volume or the total payments value, suggests the arrival of bad news. Our focus is to build up the volatility forecasting models for the return series of both total payments volume and total payments value, and to examine the forecasting performance of these volatility forecasting models, which are the autoregressive conditional heteroscedasticity (ARCH) model, the generalized autoregressive conditional heteroscedasticity (GARCH) model, the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model, and the Glosten-Jagannathan-Runkle (GJRGARCH) model.

3.2.1 The ARCH model

The ARCH model introduced by Engle (1982) is the one of the first models that provides a way to model conditional heteroscedasticity in volatility. In an ARCH model, ε_t is assumed to have the following representation,

$$\varepsilon_t = h_t e_t, e_t \sim IID(0, 1), \tag{7}$$

where h_t is the volatility, and h_t^2 is the conditional variance, which is specified as,

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2, \tag{8}$$

where $\alpha_0 > 0$, and $\alpha_1, ..., \alpha_p$ are assumed to be nonnegative to guarantee that the conditional variance is positive. Such an ARCH model is denoted by ARCH(p). In the ARCH(p) model, the predictable volatility depends on the past news. The effect of a return shock i periods ago on the current volatility is governed by the parameter

α_i. Normally, we would expect that $\alpha_i < \alpha_j$ for $i > j$. That is, the older the news, the less effect it has on the current volatility. In an ARCH(p) model, old news which arrived at the market more than p periods ago has no effect at all on current volatility.

In this paper, we consider an ARCH(1) model, at the initial starting time k, the 1-step ahead forecast of h_{k+1}^2 is,

$$h_k^2(1) = \alpha_0 + \alpha_1 \varepsilon_k^2. \tag{9}$$

The 2-step ahead forecast is then given by,

$$h_k^2(2) = \alpha_0 + \alpha_1 h_k^2(1). \tag{10}$$

Repeating this procedure yields the j-step ahead forecast for h_{k+j}^2,

$$h_k^2(j) = \alpha_0 + \alpha_1 h_k^2(j-1), \tag{11}$$

where $h_k^2(0) = \varepsilon_k^2$. The strength of the ARCH(1) model is that it is able to manage to model the volatility clustering and the mean reverting characteristics. The ability to model volatility clustering and the mean reverting characteristics can be seen in the definition of the conditional variance where it is evident that a large ε_{t-1}^2 will give rise to a large h_t^2. In other words, large and small chocks tend to be followed by large and small chocks, respectively. To further increase the understanding of the ARCH(1) model's dynamics it is worth noting that the ARCH(1) model can be rewritten as an AR(1) model on the squared residuals ε_t^2.

However, the ARCH(1) model suffers from a major drawback that the conditional variance in the ARCH(1) is specified with only the squared shock as a variable and is thus not able to model the asymmetric effects of positive and negative shocks. Furthermore, the ARCH(1) model imposes restrictive intervals for the parameters if it should have finite fourth moments and is likely to over predict volatility since it responds slowly to a large and isolated shock.

3.2.2 The GARCH model

The ARCH model is extended to the GARCH model (Bollerslev, 1986). In this paper, we consider the GARCH (1, 1) model, which is specified as,

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}^2, \tag{12}$$

where $\omega > 0, \alpha \geq 0, \beta \geq 0$, and $\alpha + \beta < 1$.

The forecasts of the GARCH(1, 1) model are obtained recursively as the forecasts of an ARCH(1) model. Assume that the GARCH(1, 1) model is at the forecast origin k, the 1-step ahead forecast of h_{k+1}^2 is,

$$h_{k+1}^2(1) = \omega + \alpha\varepsilon_k^2 + \beta h_k^2 \tag{13}$$

In order to conduct multi-step ahead forecast, we can have,

$$h_{t+1}^2 = \omega + (\alpha + \beta)h_t^2 + \alpha h_t^2(e_t^2 - 1).$$

Let $t = k + 1$, then the above equation yields,

$$h_{k+2}^2 = \omega + (\alpha + \beta)h_{k+1}^2 + \alpha h_{k+1}^2(e_{k+1}^2 - 1).$$

With $E[e_{k+1}^2 - 1|F_k] = 0$, the 2-step volatility forecast is,

$$h_k^2(2) = \omega + (\alpha + \beta)h_k^2(1). \tag{14}$$

At the original forecast, the general j-step ahead forecast for h_{k+j}^2 is,

$$h_k^2(j) = \omega + (\alpha + \beta)h_k^2(j - 1), j > 1. \tag{15}$$

Repeating the substitutions for $h_k^2(j-1)$ until the j-step forecast can be written as a function of $h_k^2(1)$ gives the explicit expression for the j-step ahead forecast,

$$h_k^2(j) = \frac{\omega[1-(\alpha+\beta)^{j-1}]}{1-\alpha-\beta} + (\alpha+\beta)^{j-1} h_k^2(1). \tag{16}$$

The properties of the GARCH (1, 1) model is quite similar to that of the ARCH(1) model. The GARCH(1, 1) model is able to model the volatility clustering but it is unable to model the asymmetric effect of positive and negative returns. The GARCH(1, 1) model also imposes restrictions on the parameters to have a finite fourth moment as is the case for the ARCH(1) model. The GARCH(1, 1) model is similar to the ARCH(1) model but with the addition of lagged conditional variances avoids the need for adding many lagged squared returns as was the case for the ARCH(1) model to be able to appropriately model the volatility. This greatly reduces the number of parameters that need to be estimated. In fact, the conditional variance of GARCH(1.1) can be written as

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta(\omega + \alpha\varepsilon_{t-2}^2 + \beta h_{t-2}^2).$$

3.2.3 The EGARCH model

Both the ARCH(1) model and the GARCH(1, 1) model are able to model the persistence of volatility, the so-called volatility clustering, but they assume that positive and negative shocks have the same impacts on volatility. It is possible that for the volatility of the returns of the total payments volume or the total payments value, the innovations have an asymmetric effect on the conditional variance. Statistically, this effect occurs when an unexpected drop in the total payments volume or in the total payments value, which is bad news, increases predictable volatility more than an unexpected increase in the total payments volume or the total payments value, which is good news, of similar magnitude. This effect suggests that a symmetry constraint on the conditional variance function in the past error terms is inappropriate.

To be able to model this behavior and overcome the weaknesses of the GARCH (1, 1) model, Nelson (1991) proposes the first extension to the GARCH model, called the EGARCH, which is able to allow for asymmetric effects of positive and negative asset returns. In the EGARCH (1,1) model, ε_t is assumed to have the same representation as before, but the conditional variance now is given by,

$$log(h_t^2) = \omega + \gamma\varepsilon_{t-1} + \alpha(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta log(h_{t-1}^2). \tag{17}$$

The parameter γ captures the leverage effect. For good news ($\varepsilon_{t-1} > 0$), the impact of innovation ε_{t-1} is $(\gamma+\alpha)\varepsilon_{t-1}$ and for bad news ($\varepsilon_{t-1} < 0$), it is $(\gamma-\alpha)\varepsilon_{t-1}$. If γ becomes 0, $log(h_t^2)$ responds symmetrically to ε_{t-1}. To produce a leverage effect γ must be negative. The fact that the EGARCH(1, 1) process is specified in terms of log-volatility implies that h_t^2 is always positive and, consequently, there are no restrictions on the sign of the model parameters.

At the forecast original k, the 1-step ahead forecast of h_{k+1}^2 is,

$$log(h_k^2(1)) = \omega + \gamma\varepsilon_k + \alpha(|\varepsilon_k| - E(|\varepsilon_k|)) + \beta log(h_k^2). \tag{18}$$

Since all of the parameters at the right hand side are known at time k, the one step ahead volatility forecast is simply $h_k^2(1) = h_{k+1}^2$. The multi-step ahead volatility forecast of the EGARCH(1, 1) model is not as trivial as for the other models used in this paper, which is beyond the scope of this paper.

3.2.4 The GJRGARCH model

An alternative way of modeling the asymmetric effects of positive and negative returns is presented by Glosten, Jagannathan and Runkle (1993) and resulted in the so called GJRGARCH model. In the GJRGARCH (1, 1) model, we have,

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2(1 - I[\varepsilon_{t-1} > 0]) + \gamma\varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta h_{t-1}^2 \tag{19}$$

where $\omega > 0, \alpha \geq 0, \beta \geq 0$ and $\gamma \geq 0$ to guarantee that the conditional variance is nonegative.

Positive shocks have an impact γ on the logarithm of the conditional variance, while negative shocks have an impact α_1. Typically $\alpha > \gamma$, which imposes a larger weight for negative shocks than for positive shocks in line with the leverage effect.

At the forecast original k, the 1-step ahead forecast of h_{k+1}^2 is,

$$h_k^2(1) = \omega + \alpha Z_k^2(1 - I[Z_k > 0]) + \gamma Z_k^2 I[Z_k > 0] + \beta h_k^2. \tag{20}$$

To calculate multi-step ahead forecasts, we have,

$$h_k^2(2) = E[\omega + \alpha Z_{k+1}^2(1 - I[Z_{k+1} > 0]) + \gamma Z_{k+1}^2 I[Z_{k+1} > 0] + \beta h_{k+1}^2 | I_k]. \tag{21}$$

with $E[\varepsilon_{k+1}^2 - 1 | I_k] = 0$, the 2-step volatility forecast is

$$h_k^2(2) = \omega + (\frac{\alpha + \gamma}{2} + \beta) h_k^2(1). \tag{22}$$

The general j-step ahead forecast for h_{k+j}^2, at the original forecast $h_k^2(1)$, is

$$h_k^2(j) = \omega + (\frac{\alpha + \gamma}{2} + \beta) h_k^2(j-1). \tag{23}$$

Repeating the substitutions for $h_k^2(j-1)$ until the j-step forecast can be written as a function of

$$h_k^2(j) = \omega \sum_{i=0}^{j-2} (\frac{\alpha + \gamma}{2} + \beta)^i + (\frac{\alpha + \gamma}{2} + \beta)^{j-1} h_k^2(1). \tag{24}$$

4 Model estimation and in-sample performance

In-sample diagnostic analysis is important and can reveal useful information about possible sources of model misspecification. Once the conditional variance models are specified, the next step is to estimate the parameters in the models. Each of models is fitted to the in-sample data using the Maximum Likelihood Estimation method which provides estimations for the parameters given the data and the density function of e_t in (7).

4.1 Evaluation of in-sample fit

The main problem in evaluating the predictive ability of volatility models is that the true volatility process is not observed. [3] Under the assumption that the conditional mean of r_t follows an ARMA(1, 1) process, we evaluate the in-sample performance of the three GARCH-type models as well as the constant volatility model by evaluating their performance in capturing the observed returns of the total payments volume or the total payments value.

Using the returns series of the total payments volume or total payments value delivered and received in both the LVTS and ACSS, we estimate the following four parameter models: the ARMA(1, 1) model: the conditional mean is specified as in (5) and the error term ε_t is specified to follow a normal distribution, $N(0, \sigma^2)$; the ARMA-GARCH(1, 1) model: the conditional mean is specified as in (5) and the error term is specified as $\varepsilon_t = h_t e_t$, where $e_t \sim IID(0, 1)$ and h_t is specified as in (12); the ARMA-EGARCH(1, 1) model: the conditional mean is specified as in (5) and the error term is specified as $\varepsilon_t = h_t e_t$, where $e_t \sim IID(0, 1)$ and h_t is specified as in (17); the ARMA-GJRGARCH(1, 1) model: the conditional mean is specified as in (5) and the error term is specified as $\varepsilon_t = h_t e_t$, where $e_t \sim IID(0, 1)$ and h_t is specified as in (19).

The data are collected in three frequencies, i.e., weekly, monthly, and quarterly frequencies. We divide our data into two subsamples for each of the three frequency data. The first subsample is a sample used to estimate model parameters, and the

[3] Actually, a proxy for the realized volatility is thus needed and moreover the choice of the statistical measure, as pointed by Bollerslev, Engle and Nelson (1994), is far from clear.

second subsample is a prediction sample used to evaluate out-of-sample forecasts. For weekly data, the first subsample covers the period from the first week in January 2001 to the first week in March 2021, and the second subsample from the second week in March 2021 to the fifth week in December 2021. For the monthly data, the first subsample starts from January 2001 to February 2020, and the second subsample from March 2020 to March 2021. For the quarterly data, the first subsample covers the period from the first quarter in 2000 to the second quarter in 2021, and the second subsample from the third quarter in 2020 to the third quarter in 2021.

We evaluate the performance of the three GARCH-type models, namely, ARMA-GARCH(1, 1), ARMA-EGARCH(1, 1), and ARMA-GJR-GARCH(1, 1) models, and ARMA(1, 1) model is also evaluated for comparison purpose. The estimation is performed using the Bollerslev-Wooldridge maximum likelihood approach. [4] The adequacy of these volatility models is then checked using the sign bias, the negative size bias, and the positive size bias tests, as well as the commonly used Ljung-Box test for serial correlation in the squared normalized residuals. [5] For weekly data in the LVTS, the estimation and diagnostic results for each of these models based on the return series of the total payments volume and total payments value are presented in Tables 2 and 3, respectively. For weekly data in the ACSS, the estimation and diagnostic results for each of these models based on the return series of the total payments volume and total payments value are presented in Tables 4 and 5,

[4] Bollerslev and Wooldridge (1992) study the properties of the quasi-maximum likelihood estimator (QMLE) and related test statistics in dynamic models that jointly parameterize conditional means and conditional covariances, when a normal log-likelihood is maximized but the assumption of normality is violated. Because the score of the normal log-likelihood has the martingale difference property when the first two conditional moments are correctly specified, the QMLE has a limiting normal distribution.

[5] Engle and KG (1993) propose diagnostic tests that emphasize the asymmetry of the volatility response to news. The sign bias test considers the variable $I_{\varepsilon_{t-1}<0}$ a dummy variable, where I. is an indicator function. This test examines the impact of positive and negative returns shock on volatility not predicted by the model under consideration. The negative size bias test utilizes the variable $I_{\varepsilon_{t-1}<0}$ to investigate the different effects that large and small negative returns shocks have on volatility not predicted by the volatility model. The positive size bias test utilizes the variable $I_{\varepsilon\geq 0}\varepsilon_{t-1}$. It focuses on the different impacts that large and small positive return shocks may have on volatility, which are not explained by the volatility model.

respectively. Similarly, corresponding to the monthly and quarterly data, the relative estimation and diagnostic results are reported in Tables 6, 7, ..., 13. As a convention, the asymptotic standard errors are reported in parentheses in these tables.

The estimation results in Table 2 indicate that the ARMA(1,1) model has a lower log-likelihood than ARMA-GARCH(1, 1) model, ARMA-EGARCH(1, 1) model, and ARMA-GJRGARCH(1, 1) model, which suggests that ARMA-GARCH(1, 1) model, ARMA-EGARCH(1, 1) model, and ARMA-GJRGARCH(1, 1) model significantly improves the in-sample fit of the ARMA(1, 1) model.[6] The estimated parameter corresponding to h_{t-1}^2 term in the ARMA-GARCH (1, 1) model is significant, indicating that large changes in the volatility will affect future volatilities, i.e., the characteristic of the volatility clustering in the data. The estimated parameter corresponding to the ε_{t-1} term in the ARMA-EGARCH(1, 1) model is significant and negative, which is consistent with the hypothesis that negative return shocks cause higher volatility than positive return shocks, i.e., this result points to the presentence of a leverage effect in the return series of the total payments volume in the LVTS. Meanwhile we can also see that the standard ARMA-GARCH(1, 1) model has a lower log-likelihood than the leverage models, ARMA-EGARCH(1,1) and ARMA-GJRGARCH(1,1). Although the estimated parameter corresponding to negative shock is not significant in the ARMA-GJRGARCH(1, 1) model, the estimated parameter of the positive shock is negative, suggesting a larger weight for negative shock than for positive shock in line with leverage effect. Additionally, the coefficient of the previous time periods' volatility is significant, indicating the strong signal that the heteroscedasticity exists in the conditional variance of the returns of the total payments volume in the LVTS.

In the diagnostic checks in Table 2, for the ARMA-GARCH(1, 1) model, the size bias test, the negative size bias test, and the joint test statistics are significant. This

[6]The log-likelihood value of an ARMA model or an ARMA-GARCH type model is a way to measure the goodness of fit. The higher the value of the log-likelihood, the better a model fits a data. The log-likelihood value for a given model can range from negative infinity to positive infinity. In practice, several different models are fitted to a dataset and choose the model with the highest log-likelihood value as the model that fits the data best.

implies that these variables, $S_{t-1}^{-} = I_{\{\varepsilon_{t-1} \leq 0\}}$, $S_{t-1}^{-}\varepsilon_{t-1}$, and $(1 - S_{t-1}^{-})\varepsilon_{t-1}$ still can predict the squared normalized residual, suggesting that the ARMA-GARCH(1, 1) model is misspecified in capturing the correct impact of news on volatility. For the ARMA-EGARCH(1, 1) model, both the sign bias and negative sign bias are insignificant, suggesting that the ARMA-EGARCH(1, 1) model can be used to capture the impact of both positive and negative return shocks on volatility, and the different effects that large and small negative return shocks have on volatility. However, the positive size bias test statistic is significant, indicating that the ARMA-EGARCH(1, 1) model has some problem in capturing the impacts that large and small positive return shocks may have on the volatility. The joint test is significant, suggesting that the ARMA-EGARCH(1, 1) model is unable to capture the correct impact of news on volatility, which is consistent with the fact that the positive size bias test in this model is significant. For the ARMA-GJRGARCH(1, 1) model, the similar results of testing the impact of news on volatility are obtained. However, in terms of the Log-likelihood values, the ARMA-EGARCH(1, 1) model performs better in capturing the impact of news on volatility than the ARMA-GJRGARCH(1,1) model. The Ljung-Box statistics for the squared residuals of both the ARMA-EGARCH(1,1) model and ARMA-GJRGARCH1,1 model suggest the existence of autocorrelation in the squared residuals (hence volatility), while the Ljung-Box statistic for the ARMA-GARCH(1,1) model indicates that there does not exist autocorrelation in the squared residuals.

The estimation results of each model in Table 3 show that the ARMA(1, 1) model has a much lower log-likelihood than ARMA-GARCH(1, 1) model, ARMA-EGARCH(1, 1) model, and ARMA-GJRGARCH(1, 1) model, indicating that the three GARCH-type models significantly improve the in-sample performance of the ARMA(1,1) model. For the three GARCH-type models, the estimated coefficients on h_{t-1}^2 are positive and significant, suggesting that the returns of total payments value exhibit the volatility clustering property, large changes in total payments value tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes. This confirms the volatility clustering observed in the returns of total payments value in the LVTS. Furthermore, the estimated parameters γ in both the ARMA-EGARCH(1, 1) model and ARMA-GJRGARCH(1, 1) model are negative and significant. This confirms the leverage effects, which refers to the volatility to be negatively correlated with the returns of the total payments value. In terms of the Log-likelihood value, the ARMA-EGARCH(1, 1) model performs best. Furthermore, the Ljung-Box statistic is insignificant for the ARMA-EGARCH(1,1) model but not for both the ARMA-GARCH(1, 1) model and ARMA-GJRGARCH(1,1) model, indicating that the ARMA-EGARCH(1,1) model is able to capture the auto-correlation effects in the volatility dynamics, but both the ARMA-GARCH(1,1) model and ARMA-GJRGARCH(1,1) model are unable to do so.[7]This is consistency with the fact that the ARMA-EGARCH(1,1) model has the highest log-likelihood value.

Similar as in Table 2, the adequacy of the three volatility models is checked using the sign bias test, the negative size bias test, and the positive size bias test, as well as the Ljung-Box test for serial correlation in the squared normalized residuals. The diagnostic results for each of these three models indicate that the only model to do well in capturing both the volatility clustering and leverage effects of volatility is the ARMA-EGARCH (1,1) model, because the test statistics of the sign bias, the negative size bias, and the positive size bias for the ARMA-EGARCH (1,1) model are insignificant. The joint test statistics for both the ARMA-GARCH(1, 1) model and ARMA-GJRGARCH (1, 1) model are significant, indicating the failure of both the ARMA-GARCH (1, 1) model and the ARMA-GJRGARCH (1, 1) model in capturing the volatility dynamics.

[7]The Ljung-Box test provides a means of testing for auto-correlation within the GARCH type model's standardized residuals. If the GARCH type model has done its job there should be no auto-correlation within the residuals. The null-hypothesis of the Ljung-Box test is that the auto-correlation between the standardized residuals for a set of lags is zero. If at least one auto-correlation between the residuals for a set of lags is greater than zero, then the test statistic indicates that the null-hypothesis may be rejected.

Based on the weekly returns of both the total payments volume and the total payments value in the ACSS, Tables 4 and 5 report the estimated results of alternative models considered. Similar as in Tables 2 and 3, in both Tables 4 and 5, although the ARMA-GARCH(1,1) model performs better than the ARMA(1, 1) model due to its ability to capture the volatility clustering reflected in the analysis of the data, it has worse performance than both the ARMA-EGARCH(1,1) model and ARMA-GJRGARCH (1, 1) model, indicating the goodness of fit of both the ARMA-EGARCH(1,1) model and ARMA-GJRGARCH (1, 1) model dramatically are further improved. In view of the fact that the ARMA-GARCH(1,1) model is unable to capture the asymmetric effect, this suggests that leverage effects reflected in both the ARMA-EGARCH(1, 1) model and the ARMA-GJRGARCH(1, 1) model are an important feature for modelling volatility dynamics in the returns of both total payments volume and total payments value in the ACSS. Actually, the results of the diagnostic tests reported in both Tables 4 and 5 suggest that both the ARMA-EGARCH(1,1) model and the ARMA-GJRGARCH (1, 1) model can capture the asymmetry of the volatility response to positive and negative return shocks.

It is observed in Tables 6 and 7 that the three volatility models perform better than the ARMA(1, 1) model in the light of Log Likelihood criteria, while the ARMA-EGARCH(1,1) model has better performance than the ARMA-GARCH(1, 1) model and the ARMA-GJRGARCH (1, 1) model. This can be explained by the fact that the estimated coefficients of both h_{t-1}^2 and ε_t in ARMA-EGARCH(1, 1) model are significant and have correct signs, which makes the ARMA-EGARCH(1, 1) model be able to capture the feature of both volatility clustering and leverage effects in the returns of payments volume and payments value. Note that although the estimated ARMA-GJR-GARCH(1, 1) model shows its ability to capture the leverage effects , i.e., the estimated parameter γ is greater than the estimated parameter α, the estimated parameter β is not significant, suggesting that the feature of the volatility clustering reflected in the data cannot be captured by the ARMA-GJRGARCH(1,

1) model. Estimated results in Tables 3 and 4 show that the three volatility models improve the fit of the ARMA(1, 1) model, which is consistent with the estimated results in Tables 6 and 7. This indicates that it is important to model conditional heteroscedasticity through GARCH-type models with level effect or/and leverage effect. The diagnostic test results of the impact of news on volatility strongly suggest the presence of time-varying volatility. The value of ε_{t-1} influences current volatility, positive return shocks appear to increase volatility regardless of the size, while large negative return shocks cause more volatility than small ones.

Tables 8 and 9 report the estimated results using the monthly returns of total payments volume and total payments values in ACSS, respectively. In terms of Log Likelihood Criteria, the ARMA-EGARCH(1, 1) performs the best. It is noticeable that for the monthly return of the total payments volume and total payments value in ACSS, the ARMA(1,1) model has better performance than the ARMA-GARCH(1,1) model. The failure of the ARMA-GARCH(1,1) model to capture the time-varying volatility is probability due to the fact that the volatility of the monthly returns in the ACSS is not so volatile to be captured by the GARCH(1,1) volatility model. The quadratic function is dominated by the exponential for the small ε_t.

For the quarterly returns of total payments volume and total payments value in the LVTS, the estimated results in Tables 10 and 11 indicate that the ARMA(1, 1) model performs the worst, while the ARMA-EGARCH(1, 1) has the best performance. For the estimated ARMA-EGARCH(1,1) model, the estimated coefficients of both the volatility clustering and leverage effect are significant and have the expected sign. In contrast, in the ARMA-GJRGARCH(1, 1) model the estimated coefficient of the leverage effect is insignificant (Table 10) and the estimated coefficient of the squared error (ε_{t-1}^2) is insignificant (Table 11), indicating that both the volatility clustering and the leverage effect are important for modeling the return dynamics of both total payments volume and total payments value in the LVTS. Regarding the diagnostic tests, the Ljung-Box test statistics suggest that the three GARCH-type models perform well in capturing the time-varying volatility.

In Tables 12 and 13, the three GARCH-type models perform better than the ARMA(1, 1) model. In particular, the ARMA-EGARCH (1, 1) perform best.

To sum up, our in-sample analysis of the basic structure of the modeling framework reveals some important stylized facts for the returns of both total payments volume and total payments value:

1. The in-sample analysis provides the empirical evidence of the existence of volatility clustering and leverage effects for the return series of both total payments volume and total payments value in the LVTS and ACSS. This indicates that it is important to model conditional heteroscedasticity through the GARCH-type models. In general, the three GARCH-type model perform better than the ARMA(1, 1) model with constant volatility.

2. Among all the three GARCH-type models, the ARMA-EGARCH(1, 1) model performs best to estimate and forecast the three different frequency return series, i.e., weekly, monthly, and quarterly returns. This indicates that not only does volatility clustering plays an important role in forecasting the volatility of the returns of the total payments volume or the total payments value, but also asymmetrical effects of return shocks is a critical factor in capturing the volatility dynamics of the returns of the total payments volume or the total payments value.

5 Out-of-sample performance

The foregoing in-sample analysis demonstrates that modeling volatility clustering and leverage effects through ARMA-EGARCH(1,1) model significantly improves the in-sample fit of historical total payment transactions data. However, it is not clear whether ARMA-EGARCH (1, 1) model will also perform well in out-of-sample forecast for volatility. Even if a model has been chosen and fitted to the data and the volatility forecasts have been calculated, evaluating the forecasting performance of the volatility is difficult due to the latent nature of volatility, i.e., the volatility is unobservable.

In this section, we evaluate the out-of-sample predictive ability of the ARMA-EGARCH(1,1) model. In particular, we are interested in comparing the relative out-of-sample predictive accuracy between the ARMA-EGARCH(1, 1) model and the ARMA(1, 1) model with constant volatility. Therefore, we measure the relative out-of-sample forecasting performance according to the ratio of the mean squared error (RMSE), the ratio of the mean absolute error (RMAE), the ratio of the mean absolute percent error (RMAPE) of the ARMA-EGARCH (1,1) model relative to the ARMA(1, 1) model with constant volatility.

The RMSE of ARMA-EGARCH(1, 1) model relative to the ARMA(1, 1) model with constant volatility:

$$\text{RMSE} = \frac{\sum_{t=n}^{n+p}(r_t - \hat{r}_t^{\text{ARMA-EGARCH(1,1)}})^2}{\sum_{t=n}^{n+p}(r_t - \hat{r}_t^{ARMA(1,1)})^2}, \tag{25}$$

where $\hat{r}_t^{\text{ARMA-EGARCH(1, 1)}}$ and $\hat{r}_t^{\text{ARMA(1, 1)}}$ express the forecasted values of r_t by the ARMA-EGARCH(1, 1) model and ARMA(1, 1) model with constant volatility, respectively.

When the value of the RMSE is above one, it indicates that the forecast performance of the ARMA-EGARCH(1, 1) model is worse than that of the ARMA(1, 1) model. When it is below one, the forecast performance of the ARMA-EGARCH(1, 1) model is better than that of the ARMA(1, 1) model.

RMAE of ARMA-EGARCH(1, 1) relative to the ARMA (1, 1) model with constant volatility:

$$\text{RMAE} = \frac{\sum_{t=n}^{n+p}|(r_t - \hat{r}_t^{\text{ARMA-EGARCH(1, 1)}})|}{\sum_{t=n}^{n+p}|(r_t - \hat{r}_t^{\text{ARMA(1, 1)}})|}. \tag{26}$$

If the value of RMAD is above one, the forecasting performance of the ARMA-

EGARCH(1, 1) model is worse than that of the ARMA(1, 1) model. When the value of RMAD is below one, ARMA-EGARCH(1, 1) model outperforms the ARMA (1, 1) model.

MAPE of the ARMA-EGARCH(1, 1) model relative to that of ARMA(1, 1) model with constant volatility:

$$\text{RMAE} = \frac{\sum_{t=n}^{n+p} |(r_t - \hat{r}_t^{\text{ARMA-EGARCH(1, 1)}})/r(t)|}{\sum_{t=n}^{n+p} |(r_t - \hat{r}_t^{\text{ARMA(1, 1)}})/r(t)|}. \tag{27}$$

Similarly, if the value of the MAPE is above one, the ARMA-EGARCH(1, 1) model performs worse than the ARMA(1, 1) model. If the value of the MAPE is below one, the ARMA-EGARCH(1, 1) model performs better than the ARMA(1, 1) model with constant volatility.

The results of RMSE, RMAD, and RMAPE are reported in Table 14 for the three different frequency data, i.e., weekly data, monthly data, and quarterly data. For the three different frequency data in the LVTS, since the values of RMSE, RMAD, and RMAPE are less than one in general, the ARMA-EGARCH (1, 1) model has better forecasting performance for the volatility of the return series of both total payments volume and total payments value than ARMA(1, 1) model with homoscedastic volatility.

For the weekly and monthly frequency data in the ACSS, in terms of the fact that most of the values of RMSE, RMAD, and RMAPE are less than one, in general the ARMA-EGARCH(1,1) model outperforms the ARMA(1, 1) model for forecasting volatility of return series of the total payments volume or the payments value. However, for the quarterly frequency data in the ACSS, the results show that in general, the ARMA(1,1) model with homoscedastic volatility has lower values of RMSE, RMAD, and RMAPE, indicating that the performance of volatility forecasting of the ARMA(1, 1) model with homoscedastic volatility outperforms the ARMA-EGARCH (1, 1) model in general. This result is consistent with what we find in estimating the ARMA-EGARCH (1, 1) model and ARMA (1, 1) model, that is, in general, we do not find volatility clustering and leverage effect for the quarterly return series in the ACSS. Thus, it is suggested to use the traditional ARMA(1,1) model with constant volatility for the quarterly return series of the total payments volume or total payments value in the ACSS.

6 Concluding remarks

Despite the numerous empirical work of modeling and forecasting the volatility of financial asset returns, little effort has been devoted to examining the issue of modeling and forecasting the volatility of the return series of both the total payments volume and total payments value delivered and received in Canada's main payment systems.

We have contributed to the literature by applying three GARCH-type models, the ARMA-GARCH(1,1) model, the ARMA-EGARCH (1, 1) model, and ARMA-GJRGARCH(1,1) model, to model and forecast the volatility dynamics of the return series of both the total payments volume and total payments value at the LVTS and ACSS. By comprehensively empirical analysis of the in-sample performance of the three GARCH-type models compared with the ARMA(1, 1) model, we find that all the three GARCH- type volatility models have better performance than the ARMA(1, 1) model with homoscedastic volatility, indicating that the GARCH-type volatility models significantly improve the estimation and forecast of the volatility of the returns of both payments volume and payment value. In particular, among the three GARCH-type volatility models, the ARMA-EGARCH(1,1) model performs best, suggesting that although the ARMA-GARCH(1, 1) model can improve the forecasting performance of the ARMA(1, 1) model with homoscedastic volatility because volatility clustering is an important feature of return series of the total payment transaction data, it cannot capture the asymmetric effects of the return shocks. Therefore, the ARMA-EGARCH(1,1) model can further significantly improve the in-sample forecasting performance of the ARMA-GARCH(1, 1) model.

The out-of-sample results indicate that the ARMA-EGARCH (1, 1) model has better performance for forecasting volatility of the return series of the total payment transaction data in the LVTS than the ARMA(1, 1) model with constant volatility. Similarly, for both the weekly and the monthly frequency data, the ARMA-EGARCH (1, 1) model outperforms the ARMA(1,1) model with constant volatility. However, for the quarterly frequency data in the ACSS, in general the forecasting performance of the ARMA(1, 1) model with homoscedastic volatility outperforms the ARMA-EGARCH (1, 1) model. This result is consistent with what we find in estimating the ARMA-EGARCH (1, 1) model and ARMA (1, 1) model. As a result, it is suggested to use the traditional ARMA(1,1) model with constant volatility to forecast the volatility of the quarterly returns of the total payments volume or total payments value in the ACSS.

Table 2: Estimation Results of Alternative ARMA-GARCH Models Using Weekly Return of Payments Volume (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0048	0.0010	0.0015	0.0012
		(0.0004)	(0.0001)	(0.0001)	(0.0001)
θ_1		-0.1764	-0.1761	-0.0952	-0.1433
		(0.0616)	(0.0281)	(0.0048)	(0.0245)
θ_2		-0.8718	-0.9914	-0.9903	-0.9888
		(0.0204)	(0.0000)	(0.0000)	(0.0000)
ω			0.0001	-0.7101	0.01511
			(0.0000)	(0.0232)	(0.0000)
α			0.0000	0.6452	0.0000
			(0.0001)	(0.0326)	(0.0004)
β			0.9990	0.6989	0.9204
			(0.0000)	(0.0044)	(0.0070)
γ				-0.1556	-0.0601
				(0.0354)	(0.0001)
		Diagnostic Test Results			
Log-likelihood		-444.80	-440.34	-221.54	-364.72
Ljung-Box Test			9.485	4.414	8.642
Sign Bias			2.062	0.367	0.9595
Negative Size Bias			2.115	0.6528	1.3164
Positive Size Bias			6.011	2.240	5.4241
Joint Effect			61.87	6.3303	55.9338

In Table 2, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 3: Estimation Results of Alternative ARMA-GARCH Models Using Weekly Return of Payments Value (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA) EGARCH	ARMA GJRGARCH
c		9e-04	0.0009	0.0045	0.0014
		(5e-04)	(0.0002)	(0.0002)	(0.0000)
θ_1		-0.0807	-0.0805	-0.0775	-0.0979
		(0.0285)	(0.0283)	(0.0052)	(0.0074)
θ_2		-09527	-0.9532	-0.9304	-0.9570
		(0.0803)	(0.0037)	(0.0000)	(0.0024)
ω			0.0001	-0.7028	0.0122
			(0.0003)	(0.0068)	(0.0000)
α			0.0000	0.3331	0.0195
			(0.0021)	(0.0127)	(0.0000)
β			0.9990	0.6761	0.9388
			(0.0001)	(0.0000)	(0.0017)
γ				-0.4515	-0.04836
				(0.0526)	(0.0001)
	Diagnostic Test Results				
Log-likelihood		-604.66	-603.60	-447.95	-573.22
Ljung-Box test			9.568	4.974	8.071
Sign Bias			1.4521	0.6553	1.4634
Negative Size Bias			1.9088	0.0589	1.9176
Positive Size Bias			0.2624	0.3519	0.2125
Joint Test			18.7141	0.5762	18.634

In Table 3, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 4: Estimation Results of Alternative ARMA-GARCH Models Using Weekly Return of Payments Volume (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA(1,1) GJRGARCH
c		8e-04	0.0007	0.0029	0.0025
		(2e-04)	(0.0001)	(0.0000)	(0.0000)
θ_1		-0.2166	-0.2171	-0.1624	-0.1541
		(0.0279)	(0.0278)	(0.0086)	(0.0005)
θ_2		-0.9853	-0.9858	-0.9392	-0.9702
		(0.1381)	(0.0000)	(0.0000)	(0.0001)
ω			0.0002	-1.1793	0.0379
			(0.0000)	(0.0015)	(0.0001)
α			0.0000	0.3138	0.2268
			(0.0002)	(0.0070)	(0.0002)
β			0.9989	0.4381	0.8176
			(0.0001)	(0.0008)	(0.0003)
γ				-0.7109	-0.2987
				(0.0313)	(0.0002)
		Diagnostic Test Results			
Log-likelihood		-742.22	-739.17	-549.50	-609.24
ARCH-LM Test			8.293	3.675	6.846
Ljung-Box test			12.34	0.3404	8.549
Sign Bias			0.8261	1.1495	2.5593
Negative Size Bias			0.9789	0.0674	0.5989
Positive Size Bias			3.2663	1.1983	0.5627
Joint Effect			47.3130	10.1024	27.6161

In Table 4, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 5: Estimation Results of Alternative ARMA-GARCH Models Using Weekly Return of Payments Value (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoskedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		4e-04	0.0004	0.0021	0.0003
		(2e-04)	(0.0001)	(0.0000)	(0.0000)
θ_1		-0.3153	-0.3158	-0.2526	-0.2676
		(0.0271)	(0.0271)	(0.0000)	(0.0019)
θ_2		-0.9778	-0.9776	-0.9626	-0.9762
		(0.0941)	(0.0001)	(0.0000)	(0.0001)
ω			0.0002	-0.8724	0.0136
			(0.0001)	(0.0000)	(0.0000)
α			0.0000	0.4017	0.2046
			(0.0007)	(0.0001)	(0.0005)
β			0.9990	0.6262	0.9012
			(0.0000)	(0.0000)	(0.0024)
γ				-0.6304	-0.2562
				(0.0254)	(0.0011)
		Diagnostic Test Results			
Log-likelihood		-595.82	-593.22	-365.79	-538.71
ARCH-LM Test			12.20	5.839	6.558
Ljung-Box Test			14.64	6.207	8.303
Sign Bias			3.039	1.949	3.380
Negative Size Bias			1.0020	2.090	0.697
Positive Size Bias			0.816	0.668	0.828
Joint Effect			47.594	5.520	32.035

In Table 5, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 6: Estimation Results of Alternative ARMA-GARCH Models Using Monthly Return of Payments Volume (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0048	0.0047	0.0048	0.0047
		(0.0004)	(0.0004)	(0.0005)	(0.0004)
θ_1		-0.1764	-0.2130	-0.2109	-0.2276
		(0.0616)	(0.0734)	(0.0780)	(0.0633)
θ_2		-0.8718	-0.8677	-0.8329	-0.8589
		(0.0204)	(0.0406)	(0.0651)	(0.0564)
ω			0.0004	-0.9461	0.0004
			(0.0004)	(0.0181)	(0.0016)
α			0.0169	-0.1107	0.0000
			(0.0208)	(0.0630)	(0.3000)
β			0.8611	0.8370	0.8439
			(0.1531)	(0.0015)	(0.7022)
γ				-0.2671	0.0391
				(0.0202)	(0.2125)
		Diagnostic Test Results			
Log-likelihood		358.11	361.09	362.53	362.14
ARCH-LM Test			6.498	10.59	4.664
Ljung-Box Test			0.521	0.512	0.760
Sign Bias			1.2064	1.4622	1.2299
Negative Size Bias			0.1638	0.1677	0.2973
Positive Size Bias			1.7211	1.2446	1.7633
Joint Effect			3.3549	3.5575	3.7273

In Table 6, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2(1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 7: Estimation Results of Alternative ARMA-GARCH Models Using Monthly Return of Payments Value (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0073	0.0047	0.0047	0.0041
		(0.0032)	(0.0018)	(0.0018)	(0.0019)
θ_1		-0.1862	-0.2379	-0.2349	-0.2392
		(0.0621)	(0.0795)	(0.0478)	(0.0456)
θ_2		-0.3679	-0.5325	-0.5219	-0.5198
		(0.0886)	(0.0635)	(0.1111)	(0.0713)
ω			0.0053	-1.8504	0.0043
			(0.0022)	(0.9709)	(0.0021)
α			0.3207	-0.0010	0.2492
			(0.1258)	(0.0756)	(0.1533)
β			0.0000	0.6197	0.1344
			(0.3368)	(0.1817)	(0.3323)
γ				0.4823	0.1690
				(0.1192)	(0.2099)
		Diagnostic Test Results			
Log-likelihood		239.21	262.29	266.84	264.12
ARCH-LM Test			5.535	3.58	4.591
Ljung-Box Test			1.214	5.356	1.485
Sign Bias			1.3462	1.816	1.5996
Negative Size Bias			0.4093	0.3502	0.7527
Positive Size Bias			1.0757	2.0585	1.4286
Joint Effect			2.3013	5.6873	3.3025

In Table 7, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 8: Estimation Results of Alternative ARMA-GARCH Models Using Monthly Return of Payments Volume (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA(1,1) GJRGARCH
c		0.0029	0.0031	0.0036	0.0031
		(0.0004)	(0.0040)	(0.0004)	(0.0005)
θ_1		0.0258	0.0040	-0.0971	0.0027
		(0.0629)	(0.0695)	(0.0637)	(0.0691)
θ_2		-0.9188	-0.8774	-0.8759	-0.8777
		(0.1594)	(0.0288)	(0.0208)	(0.0275)
ω			0.0001	-9.0797	0.0000
			(0.0004)	(0.3542)	(0.0000)
α			0.0203	-0.3887	0.0000
			(0.0218)	(0.0516)	(0.0118)
β			0.9582	-0.6874	0.9890
			(0.1059)	(0.0586)	(0.0057)
γ				0.1374	0.0199
				(0.0887)	(0.0162)
		Diagnostic Test Results			
Log-likelihood		310.53	307.69	325.67	308.89
ARCH-LM Test			0.0531	0.4056	0.134
Ljung-Box Test			2.215	3.011	(1.806)
Sign Bias			0.8948	0.2209	0.9734
Negative Size Bias			1.7337*	0.8310	1.6308
Positive Size Bias			0.8708	1.1753	0.7970
Joint Effect			18.6694**	2.0962	17.9450**

In Table 8, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2(1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 9: Estimation Results of Alternative ARMA-GARCH Models Using Monthly Return of Payments Value (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0017	0.0018	0.0013	0.0019
		(0.0011)	(0.0010)	(0.0000)	(0.0010)
θ_1		0.2196	0.2301	0.1661	0.2263
		(0.0618)	(0.0801)	(0.0022)	(0.0807)
θ_2		-0.8245	-0.8278	-0.8145	-0.8234
		(0.1355)	(0.0447)	(0.0043)	0.0453
ω			0.0001	-0.2339	0.0000
			(0.0001)	(0.0000)	0.0000
α			0.0000	-0.0217	0.0000
			(0.0181)	(0.0044)	0.0016
β			0.9853	0.9541	1.0000
			(0.0068)	(0.0000)	0.0000
γ				-0.1420	-0.0122
				(0.0000)	0.0119
		Diagnostic Test Results			
Log-likelihood		299.06	298.77	301.71	299.32
ARCH-LM Test			0.8866	1.070	0.8652
Ljung-Box Test			1.360	5.022	1.881
Sign Bias			1.0488	1.105	0.9314
Negative Size Bias			0.7793	1.362	0.9769
Positive Size Bias			0.9464	1.527	0.9218
Joint Effect			4.7043	7.677	5.1794

In Table 9, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 10: Estimation Results of Alternative ARMA-GARCH Models Using Quarterly Return of Payments Volume (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0144	0.0134	0.0121	0.0136
		(0.0019)	(0.0027)	(0.0000)	(0.0019)
θ_1		-0.4407	-1.0000	-0.9588	-0.4238
		(0.1402)	(0.0178)	(0.0001)	(0.2988)
θ_2		-0.2237	0.9395	0.8065	-0.0447
		(0.2789)	(0.0577)	(0.0001)	(0.6472)
ω			0.0006	-3.0656	0.0001
			(0.0001)	(0.0002)	(0.0002)
α			0.0000	-0.0587	0.0000
			(0.0222)	(0.0000)	(0.0800)
β			0.2015	0.5747	0.8500
			0.1472	(0.0000)	(0.4555)
γ				-1.1892	0.0727
				(0.0001)	(0.0575)
		Diagnostic Test Results			
Log-likelihood		167.06	176.40	183.45	171.12
ARCH-LM Test			0.0344	0.1533	0.1158
Ljung-Box Test			0.0701	0.0897	0.1113
Sign Bias			0.0811	0.5060	2.410*
Negative Size Bias			0.7410	0.2222	1.205
Positive Size Bias			0.6387	0.4369	1.090
Joint Effect			0.9789	0.9538	6.047

In Table 10, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 11: Estimation Results of Alternative ARMA-GARCH Models Using Quarterly Return of Payments Value (LVTS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0256	0.0103	0.0147	0.0086
		(0.0158)	(0.0050)	(0.0001)	(0.0056)
θ_1		0.8038	-0.8308	-0.8127	-0.8606
		(0.0153)	(0.1931)	(0.0011)	(0.2376)
θ_2		-0.6635	0.6597	0.6412	0.7055
		(0.0232)	(0.2657)	(0.0009)	(0.3600)
ω			0.0006	-0.6019	0.0002
			(0.0009)	(0.0010)	(0.0001)
α			0.0963	0.0783	0.0000
			(0.2018)	(0.0001)	(0.0379)
β			0.7443	0.8964	1.0000
			(0.3253)	(0.0030)	(0.0000)
γ				-0.5748	-0.2366
				(0.0010)	(0.0456)
		Diagnostic Test Results			
Log-likelihood		84.66	118.21	123.29	118.29
ARCH-LM Test			0.0035	0.0139	0.0016
Ljung-Box Test			0.1703	0.0065	0.0986
Sign Bias			1.0588	0.8950	1.2217
Negative Size Bias			0.0993	0.0889	0.0489
Positive Size Bias			3.4193	3.3432**	3.5403**
Joint Effect			12.8474	12.5536**	13.5202**

In Table 11, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 12: Estimation Results of Alternative ARMA-GARCH Models Using Quarterly Return of Payments Volume (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0089	0.0149	0.0205	0.0146
		(0.0010)	(0.0842)	(0.0000)	(0.0000)
θ_1		-0.1982	-0.2576	-0.2407	-0.1991
		(0.1235)	(0.3319)	(0.0000)	(0.0015)
θ_2		-0.7837	-0.5886	-0.4396	-0.6116
		(0.2106)	(2.8743)	(0.0001)	(0.0023)
ω			0.0000	-4.1507	0.0028
			(0.0028)	(0.0007)	(0.0000)
α			0.0000	1.3085	0.0000
			(10.4719)	(0.0002)	(0.0000)
β			0.9990	0.3205	0.7382
			(6.0052)	(0.0000)	(0.0022)
γ				-2.1611	-0.8952
				(0.0004)	(0.0028)
		Diagnostic Test Results			
Log-likelihood		132.15	132.92	161.43	147.43
ARCH-LM Test			2.141	0.0016	0.9013
Ljung-Box Test			0.2765	0.0000	0.7778
Sign Bias			0.5900	0.5071	0.0663
Negative Size Bias			0.5490	2.6239**	0.3027
Positive Size Bias			0.1033	0.0277	0.2908
Joint Effect			0.4227	8.0137**	0.3214

In Table 12, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 13: Estimation Results of Alternative ARMA-GARCH Models Using Quarterly Return of Payments Value (ACSS)

Parameter |	Mean: Volatility:	ARMA Homoscedasticity	ARMA GARCH	ARMA EGARCH	ARMA GJRGARCH
c		0.0035	0.0037	0.0081	0.0043
		(0.0034)	(0.0035)	(0.0000)	(0.0036)
θ_1		-0.0753	-0.0784	0.3690	-0.1104
		(0.1123)	(0.1692)	(0.0000)	(0.1780)
θ_2		-0.4902	-0.4956	-0.6156	-0.4387
		(0.1802)	(0.1348)	(0.0001)	(0.1643)
ω			0.0000	-3.4638	0.0001
			0.0003()	(0.0004)	(0.0001)
α			0.0000	-0.8752	0.0000
			(0.0616)	(0.0001)	(0.0088)
β			0.9884	0.4045	1.0000
			(0.0506)	(0.0000)	(0.0001)
γ				-2.0859	-0.0400
				(0.0002)	(0.0431)
		Diagnostic Test Results			
Log-likelihood		108.83	108.92	123.28	109.45
ARCH-LM Test			0.390	0.4659	0.4328
Ljung-Box Test			1.038	3.716	0.900
Sign Bias			2.240**	1.1623	2.300**
Negative Size Bias			2.529**	0.2489	2.467**
Positive Size Bias			1.584	0.3278	1.772*
Joint Effect			17.329**	1.9703	19.169**

In Table 13, all models are estimated using the Bollerslev-Wooldridge maximum likelihood approach. The four models are specified as : $r_t = \alpha_0 + \alpha_1 r_{t-1} - \beta_1 \varepsilon_{t-1} + \varepsilon_t$, and $\varepsilon_t = h_t e_t$. For ARMA model with homoscedasticity, $h_t = 1$; for ARMA-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; for ARMA-EGARCH (1, 1) model, $log(h_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 log(h_{t-1}^2)$; for ARMA-GJR-GARCH(1, 1) model, $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I[\varepsilon_{t-1} > 0]) + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} > 0] + \beta_1 h_{t-1}^2$.

Table 14: Out-of-Sample Forecasting Performance

	RMSE	RMAD	RMAPE	RMSE	RMAD	RMAPE
		LVTS			ACSS	
		Return of total payments volume				
Weekly return	0.7951	0.9864	0.8926	0.8998	1.0000	0.8926
Monthly return	0.9852	0.9566	0.9737	0.9640	1.003	0.7519
Quarterly return	0.7579	0.9917	1.0016	1.0378	0.9561	0.9098
		Return of total payments value				
Weekly return	0.9895	0.9781	0.8307	1.0000	0.9939	0.9669
Monthly return	0.5797	0.7377	0.8426	0.9920	0.8905	0.9989
Quarterly return	9.005	1.0032	9.0021	1.0153	1.0004	1.1173

Table 14 reports the out-of-sample volatility forecast performance of the ARMA-EGARCH(1, 1) model. We measure the forecast performance according to the ratio of the root mean squared error (RMSE), the root mean absolute error (RMAE), the root mean absolute percent error of model ARMA-EGARCH (1,1) relative to the model ARMA(1, 1) with homoscedasticity.

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